Exercise A

Suppose $y_i \sim \text{Gamma}(\alpha, \beta_i)$, with $\mathbb{E}[y_i] = \underbrace{\frac{\alpha}{\beta_i}}_{\mu_i}$, $\text{var}(y_i) = \underbrace{\frac{\alpha}{\beta_i^2}}_{\mu_i/\beta_i}$. For large values of α, then $y_i \overset{\cdot}{\sim} N\left(\frac{\alpha}{\beta_i}, \frac{\alpha}{\beta_i^2}\right)$. Hence,

$$\log(y_i) - \log(\mu_i) \cong (y_i - \mu_i) g'(\mu_i) \Rightarrow \text{var}(g(y_i)) \approx \frac{\alpha}{\beta_i^2} \cdot g'(\mu_i)^2.$$

Using the logarithmic transformation, we get

$$\text{var}(\log(y_i)) \cong \frac{\alpha}{\beta_i^2} \cdot \frac{\beta_i^2}{\alpha^2} = \frac{1}{\alpha}, \text{ which is constant.}$$

($g'(\mu_i)^2$)