Exercise D (See also Esempio 2.4 of Salvan et al.)

The d_i contribution to the deviance is

$$d_i = 2\{ y_i[\theta(y_i) - \theta(\hat{\mu}_i)] - [b(\theta(y_i)) - b(\theta(\hat{\mu}_i))]\}$$

↖ General form

with $b(\theta_i) = -\log(-\theta_i)$, $\theta(\mu_i) = -\frac{1}{\mu_i}$

$$= 2\left\{ y_i\left[\frac{1}{\hat{\mu}_i} - \frac{1}{y_i}\right] - \left[-\log\left(\frac{1}{y_i}\right) + \log\left(\frac{1}{\hat{\mu}_i}\right)\right]\right\}$$

$$= 2\left\{ \frac{y_i}{\hat{\mu}_i} - 1 - \left[\log\left(\frac{y_i}{\hat{\mu}_i}\right)\right]\right\} = 2\left\{ \frac{y_i - \hat{\mu}_i}{\hat{\mu}_i} - \log\left(\frac{y_i}{\hat{\mu}_i}\right)\right\}$$

Hence, the deviance is

$$D(y; \hat{\mu}) = 2\sum_{i=1}^{n}\left[\frac{y_i - \hat{\mu}_i}{\hat{\mu}_i} - \log\left(\frac{y_i}{\hat{\mu}_i}\right)\right]$$