Exercise B

We will provide a very <u>informal</u> and intuitive explaination. This is <u>not a rigorous proof</u>.

The asymptotic variance of $\hat{\beta}$ is equal to

$$\mathrm{var}(\hat{\beta}) \approx (X^T W X)^{-1}, \qquad W = \mathrm{diag}(m_1 \tilde{\pi}_1 (1-\tilde{\pi}_1), \dots, m_n \tilde{\pi}_n (1-\tilde{\pi}_n)).$$

We will say that $(X^T W X)^{-1} \longrightarrow \emptyset$, i.e the std. errors get smaller and smaller if $X^T W X \longrightarrow \infty$. Note that the generic diagonal entry of such a matrix is

$$\sum_{i=1}^{n} m_i \tilde{\pi}_i (1-\tilde{\pi}_i) x_{is}^2, \qquad \text{for} \qquad s = 1, \dots, p.$$

which grows for $m_i \to \infty$ and also as $n \to \infty$ (under minor conditions on x_{is}^2).