Exercise B

The matrix X^TX is positive definite if and only if $a^T(X^TX)a > 0$, for $\forall a \in \mathbb{R}^p$ with $a \neq 0$. But then

$$a^T(X^TX)a = \underbrace{(Xa)^T(Xa)}_{b} = b^Tb = \sum_{j=1}^{p} b_j^2 > 0.$$

Note that $Xa = 0 \iff a = 0$, because $\operatorname{rank}(X) = p$.