Exercise A

We need to prove that

$$\mathbb{E}\left[\{\hat{g}(x) - g(x)\}^2\right] = \mathbb{E}\left[\hat{g}(x) - g(x)\right]^2 + \text{var}\{g(x)\}$$

Let us consider

$$\mathbb{E}\left[\{\hat{g}(x) - g(x)\}^2\right] = \mathbb{E}\left[\{\hat{g}(x) - \mathbb{E}[\hat{g}(x)] + \mathbb{E}[\hat{g}(x)] - g(x)\}^2\right]$$

$$= \underbrace{\mathbb{E}\left[\{\hat{g}(x) - \mathbb{E}[\hat{g}(x)]\}^2\right]}_{= \text{var}\{g(x)\}} + \underbrace{\mathbb{E}\left[\{g(x) - \mathbb{E}(\hat{g}(x))\}^2\right]}_{= \left[g(x) - \mathbb{E}[\hat{g}(x)]\right]^2} + \underbrace{2\,\mathbb{E}\left[\{\hat{g}(x) - \mathbb{E}[\hat{g}(x)]\}\right]}_{= 0} \times \left[g(x) - \mathbb{E}[\hat{g}(x)]\right]$$

Non-random

$$= \mathbb{E}\left[\hat{g}(x) - g(x)\right]^2$$