Exercise [redacted] A

We can prove the first equivalence using SVD: Let $X = UDV^T$

- $X\hat{\beta}_{ols} = X(X^TX)^{-1}X^Ty = UDV^T\left[VD^TDV^T\right]^{-1}(UDV^T)^Ty$

$$= U\cancel{D}\cancel{V^TV}\left[\cancel{D^TD}\right]^{-1}\cancel{V^TV}\cancel{D^T}U^Ty$$

$$= UU^Ty$$

- $Z\hat{\gamma} = Z(Z^TZ)^{-1}Z^Ty =$ Recall that $Z = XV = UD$

$$= UD\left[D^TD\right]^{-1}(UD)^Ty = U\cancel{D\left[D^TD\right]^{-1}D^T}U^Ty$$

$$= UU^Ty.$$

- $\hat{\beta}_{OLS} = (X^TX)^{-1}X^Ty = \left[VD^TDV^T\right]^{-1}(UDV^T)^Ty$

$$= V(D^TD)^{-1}\cancel{V^TV}D^TU^Ty$$

$$= V(D^TD)^{-1}D^TU^Ty.$$

- $\hat{\beta}_{pcr} = V\hat{\gamma} = V(\underbrace{Z^TZ}_{=(D^TD)})^{-1}Z^Ty = V(D^TD)^{-1}D^TU^Ty$