

Exercise C (data augmentation)

$$\tilde{X} = \begin{pmatrix} X \\ \sqrt{\lambda} I_p \end{pmatrix}; \quad \tilde{y} = \begin{pmatrix} y \\ 0_p \end{pmatrix}$$

$$(\tilde{X}^T \tilde{X})^{-1} \tilde{X}^T \tilde{y} = \left[\begin{pmatrix} X^T & \sqrt{\lambda} I_p \end{pmatrix} \begin{pmatrix} X \\ \sqrt{\lambda} I_p \end{pmatrix} \right]^{-1} \begin{pmatrix} X^T & \sqrt{\lambda} I_p \end{pmatrix} \begin{pmatrix} y \\ 0 \end{pmatrix}$$

$$= \left[X^T X + \lambda I_p \right]^{-1} (X^T y + \cancel{\sqrt{\lambda} 0}) =$$

$$= \hat{\beta}_{\text{bridge}} = (X^T X + \lambda I_p)^{-1} X^T y.$$