Exercise B

$$\hat{f}_{NW}(x) = \frac{\sum_{i=1}^{m} w_i(x) y_i}{\sum_{i=1}^{m} w_i(x)} \qquad \text{(Nadaraya-Watson)}$$

Note that $w_i(x) > 0 \quad \forall x$, therefore there are no singularity issues at the denominator.

Moreover, $w_i(x)$ is differentiable everywhere. Hence the function $\hat{f}_{NW}(x)$ is differentiable.

- If the kernel is, insted, the Epanechnikov, then

$$w_i(x) = \frac{1}{h} w\left(\frac{x - x_i}{h}\right); \qquad w(t) = \frac{3}{4}(1 - t^2)\,\mathbb{1}(|t| \leq 1)$$

Hence, because of the indicator function, the Epanechnikov kernel is not differentible for values of $t = \pm 1 \Rightarrow \left(\frac{x - x_i}{h}\right) = \pm 1$.

Hence, the estimator $\hat{f}_{NW}$ is not differentiable (but it is continuous).